AT721

HW3 Solution / Explanations

In this homework, we are asked to write code to solve the radiative transfer problem in the infrared for a perfectly emissive (black) surface, and a single homogeneous atmospheric layer. For areas in the infrared where gas absorption is minimal, this is not as far-fetched as it sounds. The ocean is very nearly black in most of the thermal infrared, and cloud layers are often treated as homogeneous (especially in retrievals).

The case is for a cloud with layer optical depth 1, single scattering albedo 0.7 and an asymmetry parameter 0.8 (typical of clouds).

Using double-Gauss quadrature, we'll discretize zenith angles with the following mu-values and weights in each hemisphere. For a 6 stream model (3 streams per hemisphere), this leads to the following quadrature set-up:

Table 1: 6 "Full-stream" quadrature scheme set-up. Double-Gauss plus the "stream with zero weight=" at the observation angle of θ=0°.

θ	$\mu = \cos\theta$	Weight	Hemisphere	Notes
0°	1.0	0.0	Upper	Target Angle
27.46°	0.8873	0.2778	Upper	
60.0°	0.5000	0.4444	Upper	
83.53°	0.1127	0.2778	Upper	
96.47°	-0.1127	0.2778	Lower	
120.0°	-0.5000	0.4444	Lower	
152.54°	-0.8873	0.2778	Lower	
180°	-1.0	0.0	Lower	Target Angle

For the given quadrature set-up, we can calculate the reflectance matrix **R** and the transmission matrix **T** for the cloud layer, where the latter includes both direct and diffuse (forward scattered) contributions. Because this is a thermal case, it is azimuthally symmetric and only the m=0 azimuthal moment need be calculated. Using the method of doubling, in addition to **R** and **T**, we also find the thermal source function for the layer upwelling & downwelling. All of these were given in the homework itself.

Because the surface is black, the principle of interaction yields a particularly simple result:

$$\mathbf{I}_{TOA}^{\uparrow} = \mathbf{R}\,\mathbf{I}_{TOA}^{\downarrow} + \mathbf{T}\,\mathbf{s}_{g}^{\uparrow} + \mathbf{s}_{atm}^{\uparrow}$$

$$\mathbf{I}_{Surf}^{\downarrow} = \mathbf{T}\,\mathbf{I}_{TOA}^{\downarrow} + \mathbf{R}\,\mathbf{s}_{g}^{\uparrow} + \mathbf{s}_{atm}^{\downarrow}$$

The first term in each equation represents the contribution from the incoming cold space. At this wavelength, cold space contributes $6 \bullet 10^{-227}$ W m^{-2} μm^{-1} sr^{-1}. I.e., it may safely be ignored!!! This is only not true in the microwave, where it can impact brightness temperatures by as much as 1-2 K.

Thus we have a contribution from each term: the surface (ground) emission term), and the atmospheric emission term. I have plotted the results vs. angle for each term below (Fig 1), where I perform renormalization of the full phase matrix (keeping 50 phase function expansion coefficient terms):

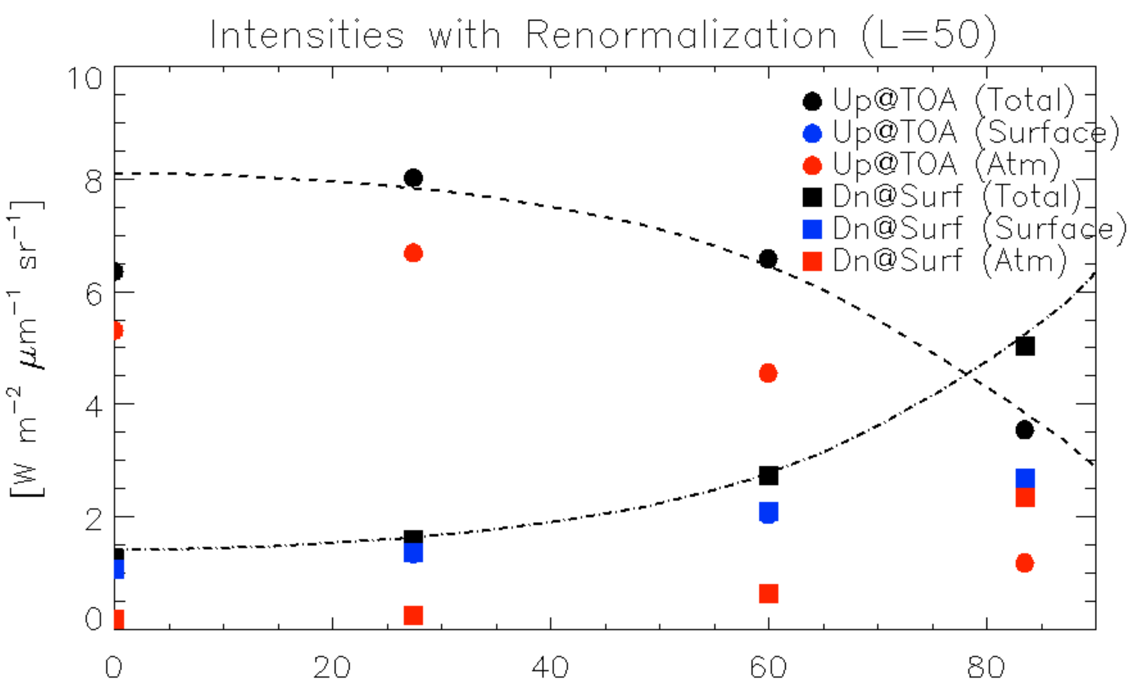

Fig 1. Calculated intensities upwelling at TOA (circles) and downwelling at the surface (squares) using a 6-stream quadrature, plus a "stream with zero weight" at the desired angle of θ=0°. Each intensity is broken down into the component from atmospheric emission (blue) and surface emission(red). Results from a very high accuracy calculation (64 streams, minimum layer optical thickness 10^{-5}) are shown in the dashed (upwelling) and dot-dash (downwellling).

It is clear that there are significant errors in our calculation, especially upwelling at TOA. It is also interesting to note that the atmospheric contribution (shown in blue) is much lower than the surface contribution (red) for the emission upwelling at TOA. This is because of the relatively large asymmetry parameter of 0.8 – while there is a fair amount of scattering, most of it is in the forward direction. This because less true right at the horizon, where the atmospheric contribution is larger. The opposite tends to be true for the downwelling surface emission, where the atmospheric contribution dominates the surface contribution. This is because only a small amount of the surface contribution is reflected off the atmospheric layer (again because of the large amount of forward scattering).

How can we reduce the errors in our calculation, especially at nadir where we didn't have a stream with weight? It turns out the "renormalization" step introduces significant errors, and a better way to go is to use the "auto-normalization" approach where we truncate the phase function at L = Nstreams-1. These leads to a normalized phase matrix with no need for renormalization, and is significantly more accurate (Fig 2). It is not clear in the literature why this is so.

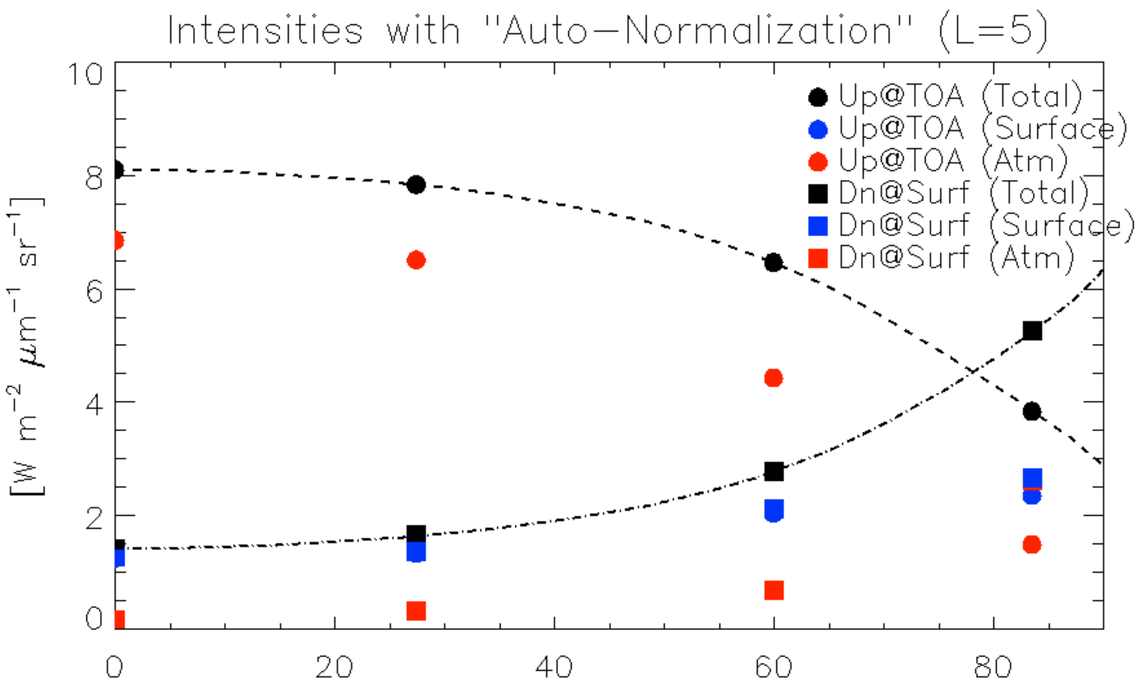

Fig 2. Same as Figure 1, but phase matrices for the 6-stream case have been calculated using the "auto-normalization" approach (L=5 in this case).

Finally, in Figure 3 I have plotted the calculated upwelling intensity at TOA at zenith (θ=0°), using both the renormalization approach and the "auto-normalization" approach. The latter is more acurrate for all numbers of streams, and generally should be used. In fact, we see that we get the correct answer to within a Kelvin with just a simple 2-stream model! It is also clear that the stream with "zero weight" is very helpful. The dashed blue line shows the result where we have extrapolated to θ=0° using a polynomial fit. For the lower numbers of streams, it tends to be significantly less accurate.

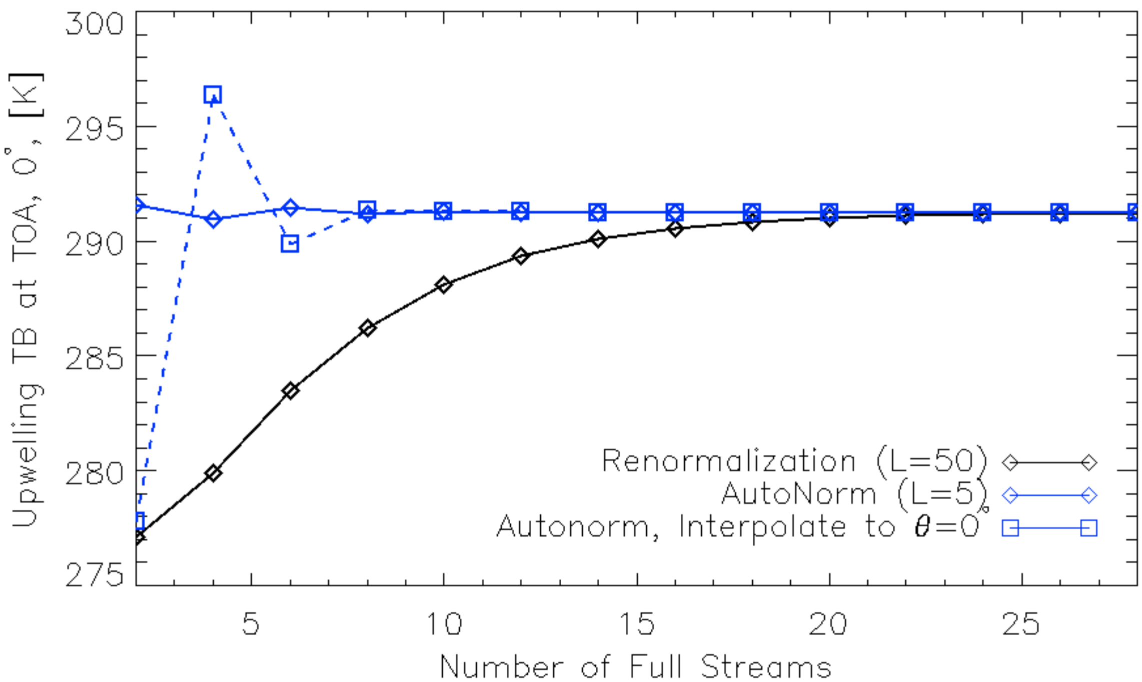

Fig 3. Three different methods for calculating upwelling intensity at TOA for zenith observations. Black shows the renormalization of the full phase function, plus the "stream with zero weight" approach. Solid blue shows the same but for the "auto-normalization" approach for calculating the phase matrices. Dashed Blue shows the "autonormalization" approach, but interpolating to find the intensity at the desired angle, rather than using the "stream with zero weight" approach.

The homework asks for the number of streams where 1% accuracy is achieved for zenith (nadir) observations. Clearly this occurs much more quickly using the "stream with zero weight" approach plus the "auto-normalization" approach for the phase function. In this case, ~0.12% accuracy is achieved immediately with a simple 2-stream model (upwelling TB error is 0.35 K, downwelling TB error 0.66 K). For the renormalization approach, it takes 12 full streams (6 per hemisphere) to achieve 1% accuracy.

Finally, we're asked to change the ssa of the layer to 0 and see how the brightness temperatures respond with no scattering. I've taken this a step farther and plotted the upwelling intensity, together with its surface and atmospheric contributions, in the figure below. The surface and atmospheric contributions generally include both non-scattered and scattered photons. One could separate the surface component using the direct vs. diffuse transmittance, but not the atmospheric component without a successive order of scattering code. In figure 4, we see that generally the upwelling TB and intensity increase with increasing atmospheric ssa. However, broken down by components, the surface contribution is increasing with ssa, while the atmospheric contribution is decreasing. This is because the layer has a very high thermal emissivity for ssa=0, but a very low emissivity as the ssa → 1.

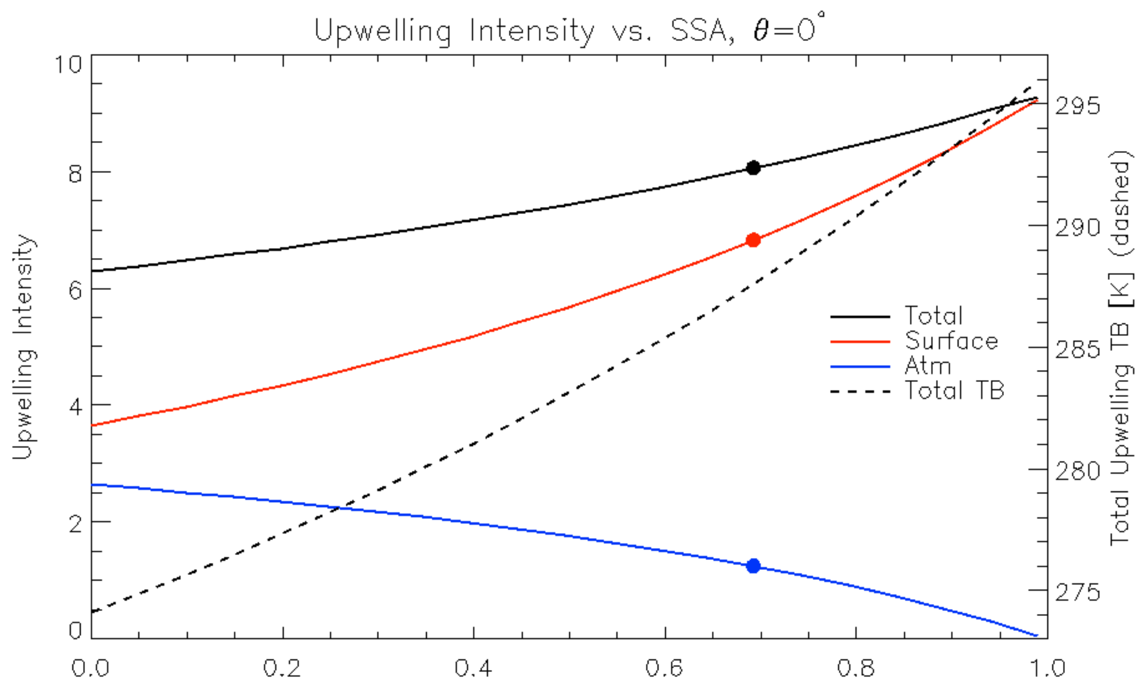

Fig 4. Upwelling Intensity and TB at TOA vs. atmospheric single scattering albedo. The observation angle is 0°. For intensity, the total (black) is shown, together with the atmospheric (blue) and surface contributions (red).

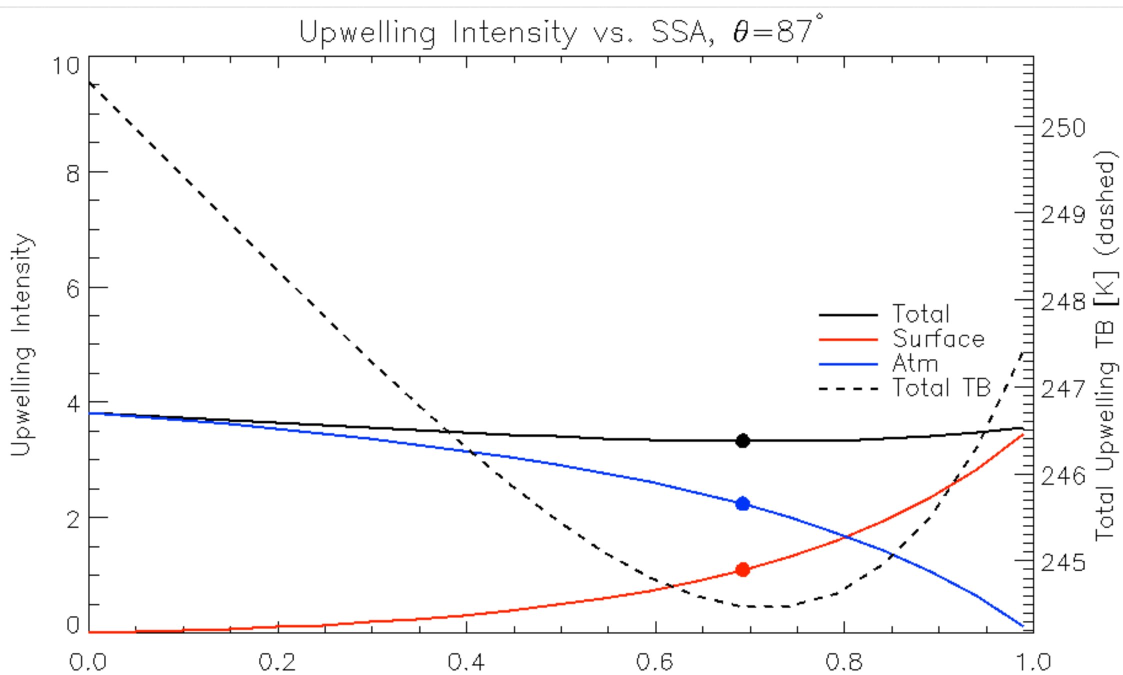

Fig 5. Same as Figure 4 but for an observation angle of 87°.