AT622 Section 5
The Sun

The main aim here is to acquaint the student with basic radiative properties of the sun and the factors that govern the disposition of solar radiation received at Earth.

5.1 The Solar Atmosphere

The sun is an entirely gaseous star composed of hydrogen (75%) and helium (25%). It is approximately 4.6 billion years old and located approximately 1.5 x 10^8 km from the Earth. It accounts for virtually all energy received by Earth and is responsible for circulation of the Earth's atmosphere and oceans. The solar atmosphere is portrayed in Fig. 5.1. The bulk of the electromagnetic radiation emitted from the sun and received at Earth arises from the vicinity of the photosphere.

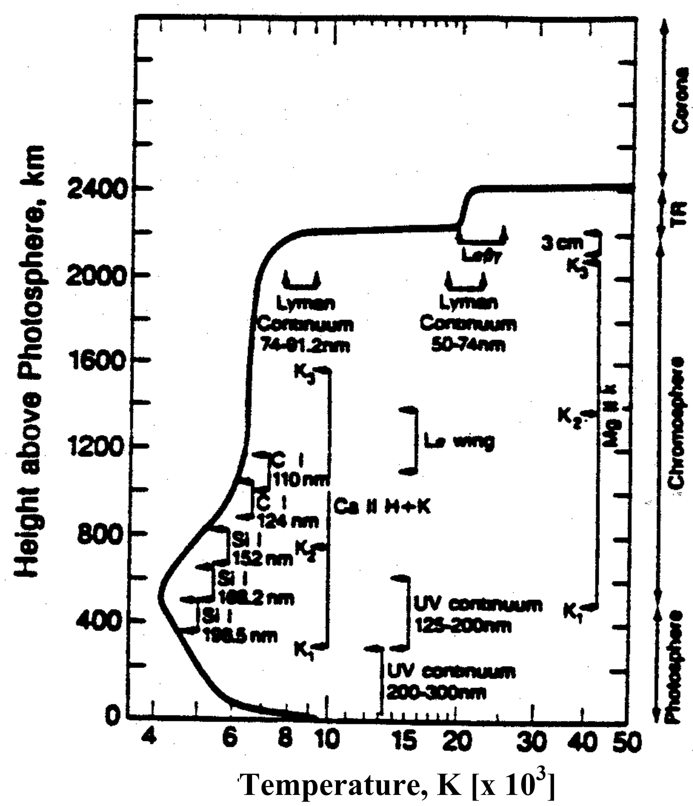


Fig. 5.1 A schematic cross section of the solar atmosphere.

The sun's emission is much like that of Earth in that it is a result of superimposing emissions from several regions within its own atmosphere. The emission from the sun is entirely analogous to the Earth's emission spectrum already shown previously where radiation arises from different levels according to the wavelength of the emission. Absorption/emission is stronger at shorter and longer wavelengths (Fig. 5.2a and c) of the solar spectrum where absorption within the atmosphere is largest (Fig. 5.2b) observe greater variability in emission due to solar activity. The emission at these extreme wavelengths originates in the rarefied corona at temperatures exceeding 10^6 K.

The intensity of solar radiation from the UV to far infrared approximately follows the 5785 K blackbody curve (Fig. 5.2a,b). The emissions at wavelengths shorter than 0.1 μm and longer than 1 cm are coronal, and highly variable. These emissions are related to measures of the solar activity, such as the sunspot number (Fig. 5.3a and b and see further discussion later). The spectrally integrated or total irradiance (i.e., the area under the curve) is a quantity that is most important for various atmospheric science applications. This irradiance measured at the top of the Earth's atmosphere under certain fixed conditions is paradoxically termed the solar constant.

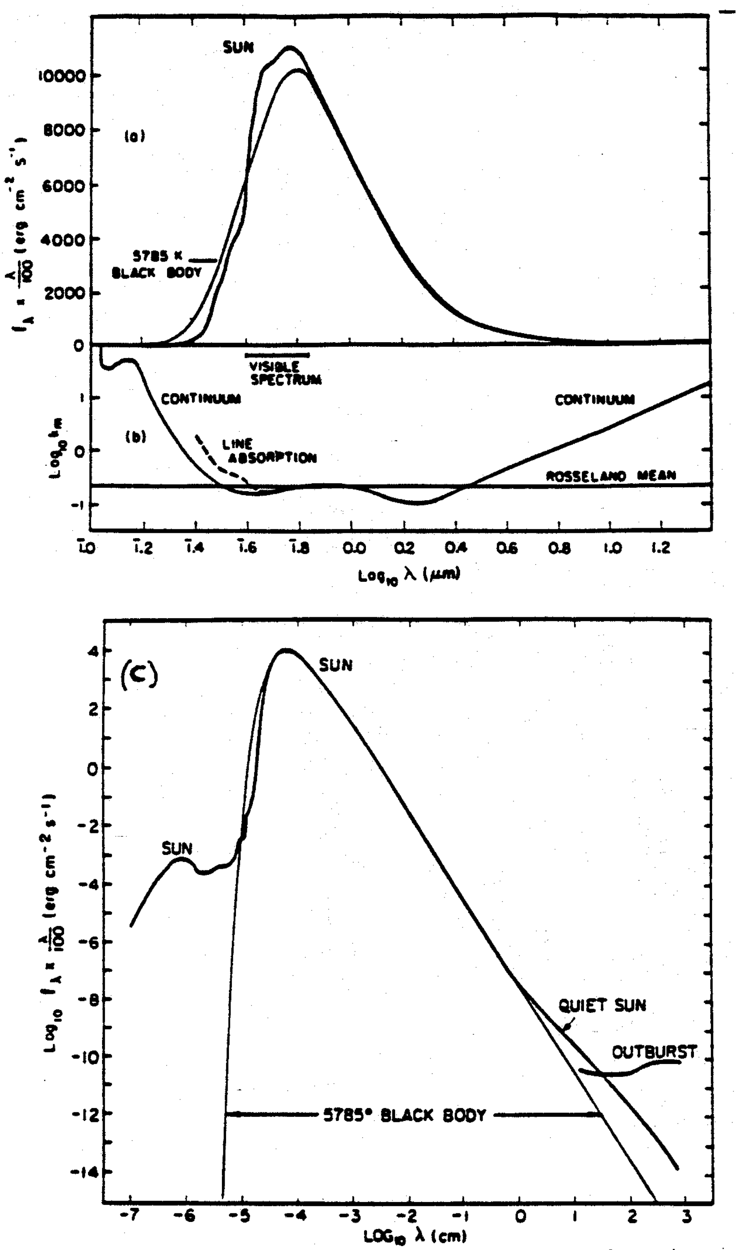


Fig. 5.2 *Spectrum of solar emission and photospheric absorption. (a) Solar spectrum compared to that of a 5784 K blackbody. The method of plotting gives areas ($f_\lambda \lambda d\ \log_{10}\lambda/100$) proportional to energy flow ($f_\lambda d\lambda$). (b) Mass absorption coefficient for the photosphere at a temperature of 5785 K. After Allen (1958). (c) The solar irradiance.*

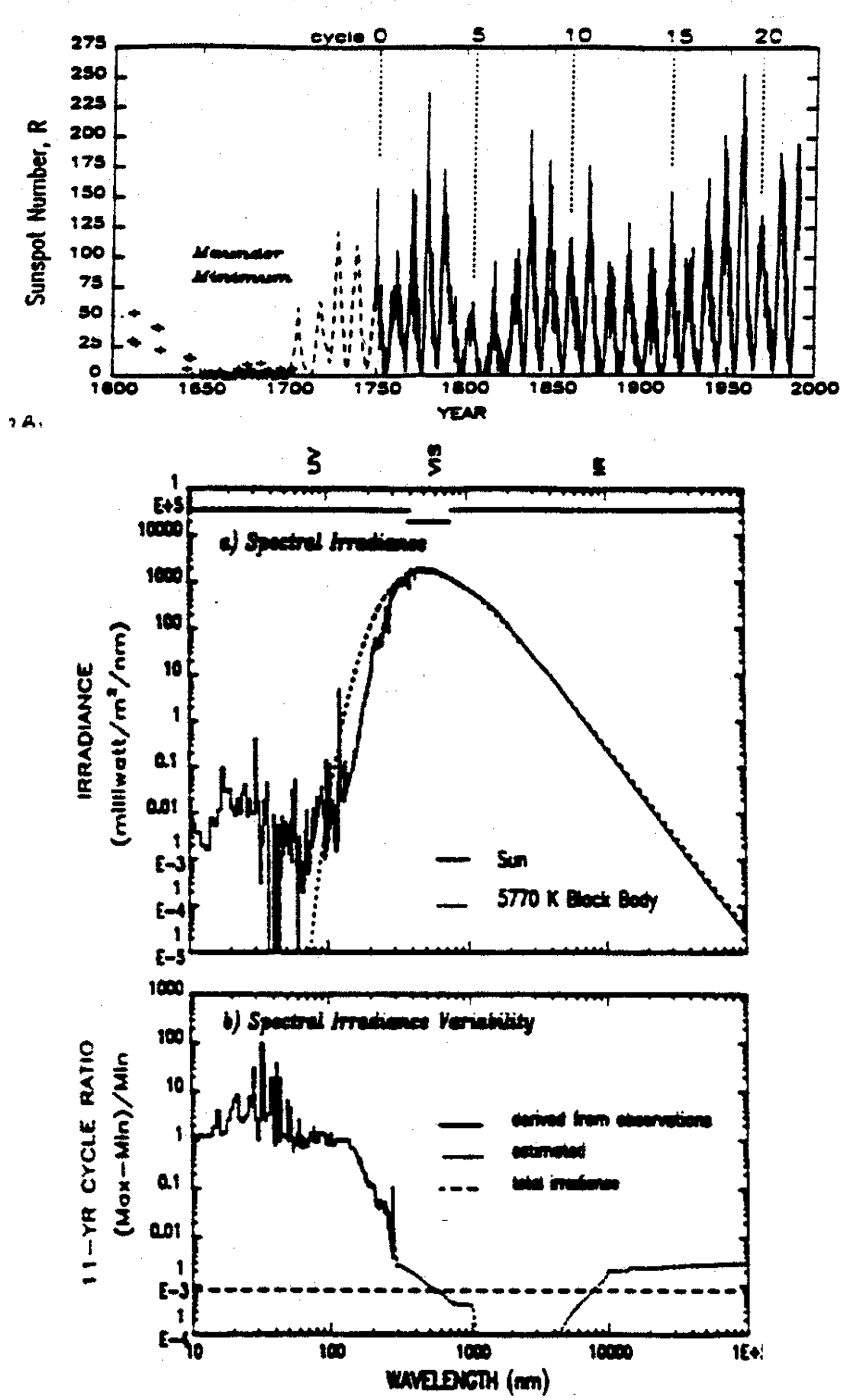


Fig. 5.3 *Solar activity as defined by the sunspot number (a) undergoes a distinct cycle, which affects the radiant output (b).*

5.2 The Solar Constant

According to our understanding of blackbody emission, a 5785 K hot body like the sun emits substantially more than a 288 K blackbody (in fact $(5785/288)^4 \approx 163{,}000$ times more). How can the Earth be in a state of radiative equilibrium: an observed equilibrium established by a balance between incoming solar and emitted longwave radiation? The answer simply lies in the dilution of the sun's radiation as it radiates out from the sun and reaches Earth.

We calculate the effect of this dilution as follows. The spectrally integrated radiation emitted by the sun and received at the top of the Earth's atmosphere is

$$Q_{\odot} = \int_0^{\infty} F_{\odot,\lambda} d\lambda \approx \frac{\sigma T_{\odot}^4}{\pi} \Omega_{\odot} \approx 1370 \text{ Wm}^{-2} \tag{5.1}$$

which, when calculated assuming the mean sun-Earth distance, is termed the *solar constant* and hereafter is denoted as $Q_{\odot}$ - **Note that by definition, the solar constant does not vary with the position of** the **Earth relative to the sun.** The *spectral* flux, defined by substituting B_λ for $\sigma T_{\odot}^4/\pi$ in Eqn. (5. 1) will be referred to as the 'spectral solar constant' $F_{\odot,\lambda}$. $Q_{\odot}$ was traditionally difficult to measure but recent instrumentation flown on satellites offer clear evidence of its variability and the magnitude of this variability.

Example 5.1: Overlapping solar and terrestrial radiation

Using both the Planck routine program, developed in Section 3, determine the fraction of the total solar radiation that falls at wavelengths below 4 μm and the fraction of radiation emitted by a 288 K blackbody at wavelengths longer than 4 μm.

```
program test
    W2=200
    W1=4
    T=288
    frac=PLANCK(W1,W2,T)
    write(*,*)'fraction=' , frac
    end
```

Output, fraction =0.998

```
program sun
    W1=0.2
    W2=4
    T= 5785
    frac= PLANCK(W1,W2,T)
    write(*,*)'fraction=' , frac
    end
```

Output, fraction =0.992

This exercise illustrates an important practical point in atmospheric physics in that only 0.8% of the total extra-terrestrial solar flux resides in wavelengths longer than 4 μm (it is actually even less than this as the energy blow of 0.2 μm is excluded in function solar). On the other hand, only about 0.2% of the total IR radiation from a 288 K blackbody resides in wavelengths shorter than 4 μm. Thus from a total energetics point of view, solar and terrestrial radiation can be treated independently due to the combination of both the dependence of blackbody radiation with temperature and the dilution of the sun's radiation as it flows to Earth.

5.3 The Solar Insolation

We will refer to the solar flux incident on a horizontal plane as solar insolation. At the top of the atmosphere, this insolation depends on the latitude, season and time of day as is expressed in the relationship

$$F_\lambda = F_{\odot,\lambda}\left(\frac{\overline{R}_{s-E}}{R_{s-E}}\right)^2 \cos\theta_\odot \tag{5.2}$$

which is the insolation at any given instant of time. R_{s-E} is the sun-Earth distance at the time of observation, $\overline{R}_{s-E}$ is the mean sun Earth distance and $\theta_\odot$ is the solar zenith angle (i.e., the angle between the local normal to Earth's surface and a line at the Earth's surface to the sun). According to this expression we acknowledge that the insolation received at the top of the atmosphere depends on

- Variations of sun-Earth distance, which in turn depends on variations in the eccentricity of the orbit of the planet around the sun.

- The sun's elevation (through $\theta_\odot$, which is influenced by astronomical factors as we will soon see). The dependence of insolation on these orbital properties was recognized by Milankovitz in his proposition that variations in these properties is the cause for ice ages on Earth.

5.4 Orbital Influence on the Insolation

Figure 5.4 illustrates the general characteristics of the Earth's orbit about the sun. The sun is situated at the focus of an ellipse and the changing Earth-sun distance as the Earth orbits around the sun, determined by the eccentricity of the orbit, creates asymmetries in solar insolation. The four reference points on this orbit, labeled 1 - 4, are the cardinal points that are used to delineate Earth's seasons.

(a) Eccentricity

The eccentricity defines the flatness of the orbital ellipse. For Mercury and Pluto, $e \sim 0.2$ and these planets are substantially closer to the sun at perihelion than at aphelion. For earth, $e \sim 0.017$ and Mars $e \sim 0.093$. In the simplest climatological sense, the summers of the southern hemisphere are hotter and winters are colder based on proximity to the sun (e.g., the Martian northern polar cap persists through summer but the southern hemisphere cap disappears).

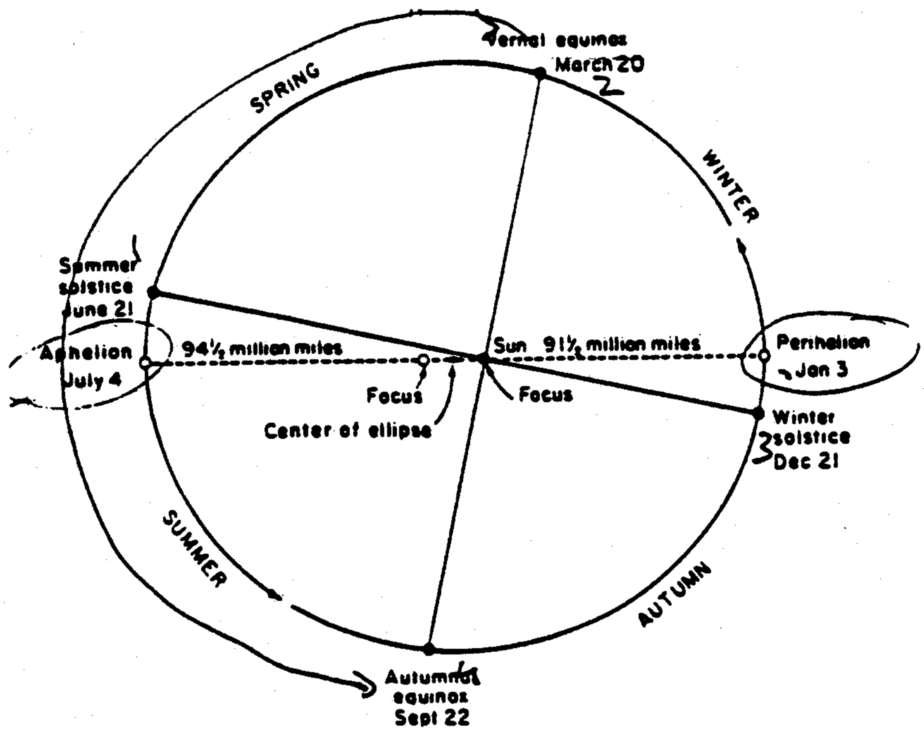


Fig. 5.4 *Dates of equinox and solstice. At the equinoxes, the Earth's axis is pointed at right angles to the sun, and day and night are of equal length all over the globe. At the summer solstice, the North Pole is tipped in the direction of the sun and the northern hemisphere has the longest day of the year. At the winter solstice, the North Pole is tipped away from the sun, and the northern hemisphere has the shortest day of the year.*

Example 5.2: The Effect of eccentricity on solar radiation received on Earth

At perihelion, $R_{s-E}/\overline{R}_{s_E} = 0.983$ and the flux received at the top of the atmosphere is

$$F_{\odot} = Q_{\odot}\left(\frac{\overline{R}_{s_E}}{R_{s-E}}\right)^2 = 1370\times1.035 = 1419$$

and at aphelion, $R_{s-E}/\overline{R}_{s_E} = 1.017$ leading to

$$F_{\odot} = Q_{\odot}\left(\frac{\overline{R}_{s_E}}{R_{s-E}}\right)^2 = 1370\times0.967 = 1325$$

Thus the amplitude of the variation of the solar insolation at the top of the atmosphere is 93 Wm^{-2}.

(b) Solar Zenith Angle (also see Sellers, Physical Climatology, p. 13-28)

The other principal factor that defines the solar insolation received by a horizontal surface at the top of the atmosphere is the solar zenith angle $\theta_\odot$.

$$\cos\theta_\odot = \sin\phi\sin\delta + \cos\phi\cos\delta\cos h \tag{5.3}$$

where ϕ is the latitude, δ is the declination and h is the hour angle.

The declination, δ, is defined as the angle formed between the equator and plane of orbit. This parameter has a substantial impact on how the solar radiation is distributed over the globe and thus on how and why seasons occur on Earth (Figs. 5.5a, b, c).

$$\delta \approx -23°27' \times \cos\left[\frac{360}{360.25} \times (JD + 9)\right]$$

where JD is Julian Day.

The hour angle, h, is defined by ±15° each hour before or after solar noon. Account must be taken for all observers not at an integral meridian.

> Denver is located at 105°W, or exactly 7 hours before GMT. Local noon occurs at 12:00. Salt Lake City is 7° further west but in the same time zone. At 12:00, the hour angle is therefore +7°, while at 1:00 p.m. the hour angle is -8°.

(c) Solar Azimuth Angle

The solar azimuth angle is given by

$$\sin\zeta = \frac{\cos\delta\sin h}{\sin\theta_\odot}$$

where ζ is referenced to the south. $\zeta > 0$ is eastward and $\zeta < 0$ is westward.

The mean total daily insolation is also a quantity of some interest in climatological studies

$F = Q_\odot \times$ fractional day length $\times \cos\bar{\theta}_\odot$

Fractional day length is determined as $2H$, and

$$\cos\bar{\theta}_\odot = \int_{\text{day length}} \cos\theta_\odot\, dt \Big/ \int dt$$

Values are tabulated in Table 5.1 and shown graphically in Figs. 5.5b and c. As we shall see, the product of the fractional day length by $\cos\theta_\odot = 1/4$ on the global average.

Figure 5.6a shows the distribution of daily insolation as a function of latitude and month. Of note are

- locations of maximum and minimum values and the relation of these to astronomical factors
- latitudes of smallest and largest seasonal variations
- asymmetrical hemispheric distribution, and
- typical values of the insolation at low, middle, and high latitudes.

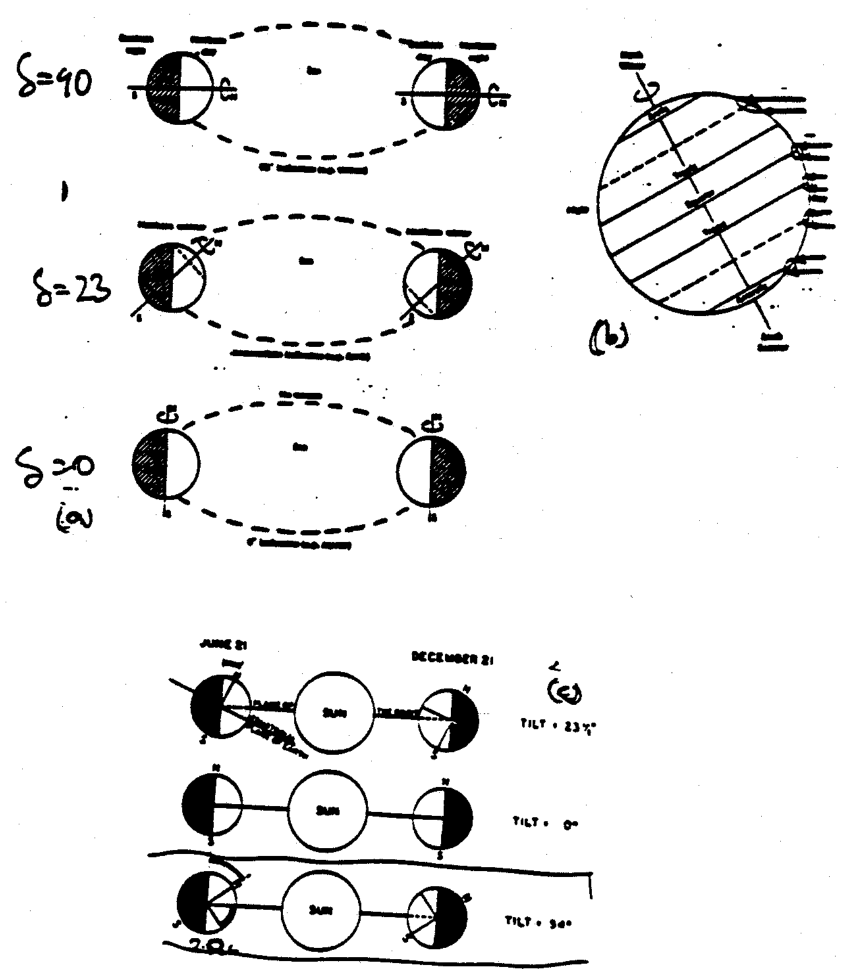


Fig. 5.5 *(a) How seasonal variations depend on the angle between the equator and the plane of the orbit. If a planet had exactly 90° inclination, it would be impossible to draw an analogy with terrestrial north and south poles. The labels in the top panel would then be arbitrary. (b) Incident solar radiation at a solstice. A beam of sunlight is spread over a larger area of ground at high latitudes, where the sun is close to the horizon, than at low latitudes where the sun is almost overhead. The day is longer than the night in the summer hemisphere whereas the night is longer than the day in the winter hemisphere. Both effects are important in determining the incident solar radiation. (c) The effect of axial tilt on the distribution of sunlight. When the tilt is decreased from its present value of 23½°, the polar regions receive less sunlight than they do today. When the tilt is increased, polar regions receive more sunlight. The possible limits of these effects (never actually achieved) would be a tilt of 0°, when the poles would receive no sunlight: and 54°, when all points on the earth would receive the same amount of sunlight annually.*

Table 5.1 The seasonal and latitudinal distributions of the length of the daytime given in parts of 24 hours and those of the weighted mean values of cos $\overline{\theta}_{\odot}$ (see Table 6 of Manabe and Moller, 1961: On the radiative equilibrium and heat balance of the atmosphere, Mon. Wea. Rev., ***8a,*** *503-532.)*

°Lat	Fractional length of daytime				Cos $\theta_{\odot}$			
	Apr.	July	Oct.	Jan.	Apr.	July	Oct.	Jan.
5	.508	.517	.500	.496	.625	.587	.614	.591
15	.521	.537	.492	.471	.618	.601	.579	.549
25	.533	.562	.483	.450	.599	.593	.524	.474
35	.546	.596	.471	.421	.558	.567	.458	.393
45	.562	.637	.454	.362	.501	.521	.379	.317
55	.596	.708	.437	.321	.423	.453	.282	.203
65	.629	.837	.404	.208	.345	.369	.176	.106
75	.750	1.000	.329	---	.241	.311	.071	---
85	1.000	1.000	---	---	.168	.318	---	---

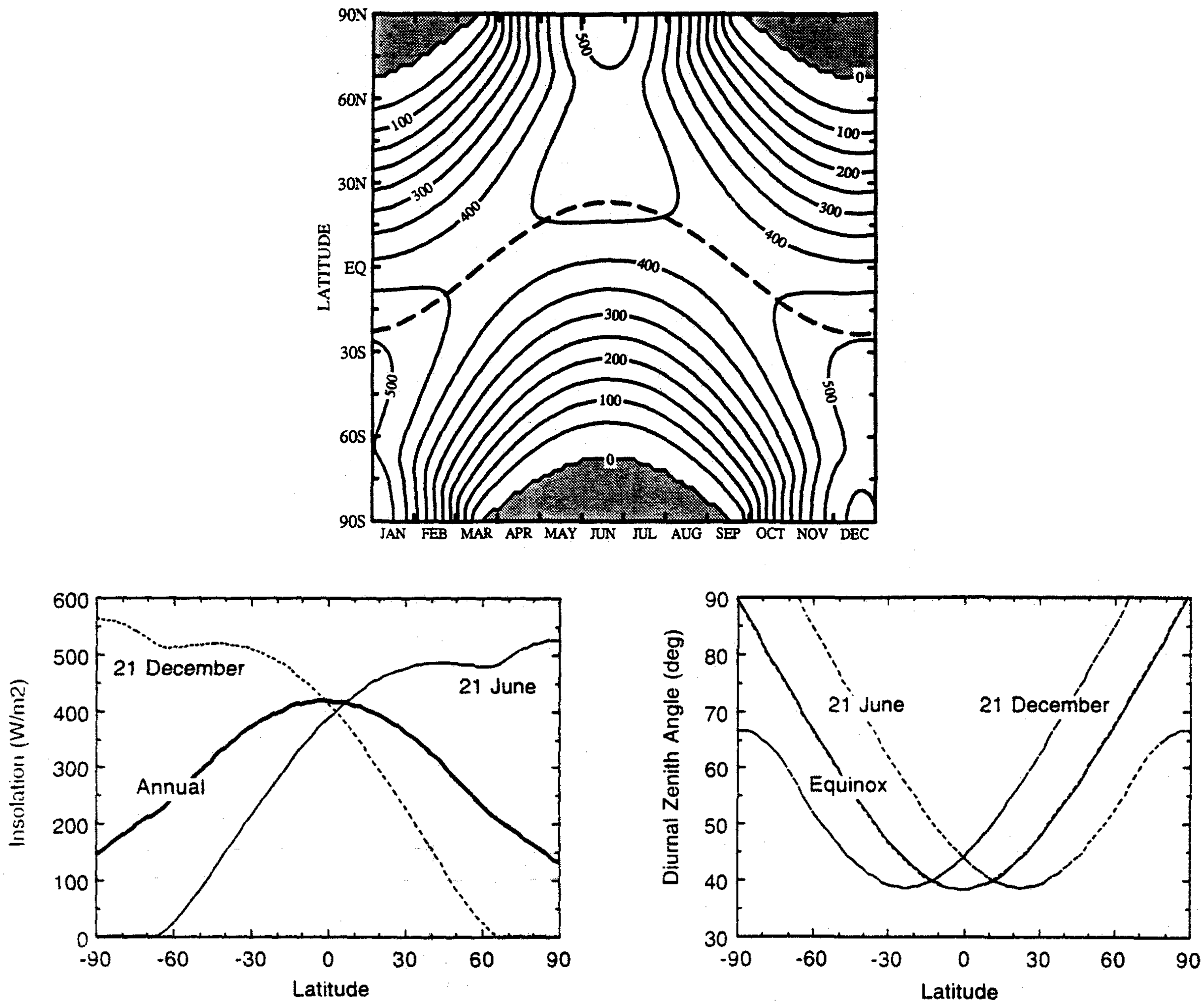


Fig. 5.6 The daily variation of the solar radiation at the top of the atmosphere as a function of latitude. The units are Wm^{-2}.

Example 5.3: Some properties of the solar zenith angle

Some examples:

- Poles: cos θ, sin ϕ, and cos $\theta_\odot$ = sin δ, 90 - $\theta_\odot$ = δ = constant, where 90 - $\theta_\odot$ is the elevation angle. Thus the sun circles the pole and is never higher than 23.5° and transition from day to night occurs at equinoxes (δ = 0).

- Solar noon: cos h = 1, $\theta_\odot = \phi - \delta$. Note since –23.5° < δ < 23.5° only for ϕ < |23.5°| can the sun be directly overhead.

- Sunrise and sunset: $h = H$ (half day length), cos $\theta_\odot$ = 0 and it follows that cos H = - tan ϕ tan δ, H = 6 hours when tan ϕ (equator) or tan δ = 0 (equinoxes).

Convenient formulae for the declination δ and the ratio $(\overline{R}_{s_E} / R_{s-E})^2$ are

$$\delta = \sum_0^3 a_n \cos n\psi_n + b_n \sin n\psi_n$$

and

$$\left(\frac{\overline{R}_{s_E}}{R_{s-E}} \right)^2 = \sum_0^3 c_n \cos n\psi_n + d_n \sin n\psi_n$$

where

$$\psi_n = \frac{2\pi \text{ day}_n}{365}$$

where the day number day$_n$ ranges from 0 on January 1 to 364 on December 31.

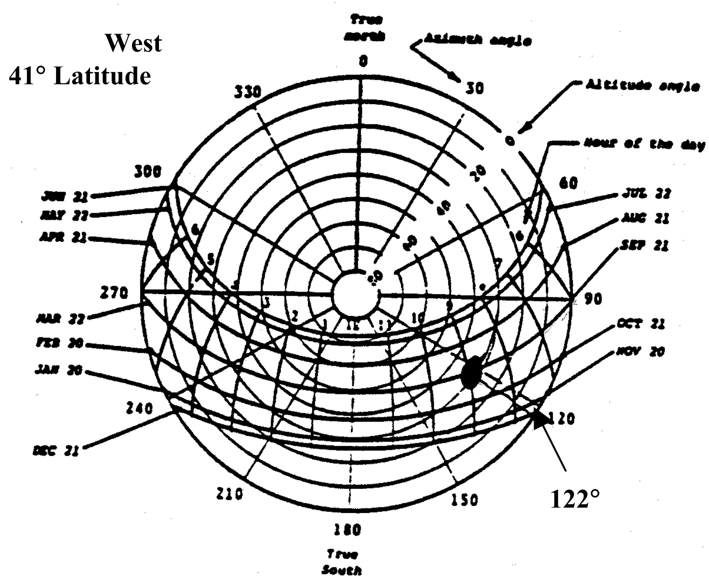
n	a_n	b_n
0	0.006918	
1	-0.399912	0.070257
2	-0.006758	0.000907
3	-0.002697	0.001480

n	c_n	d_n
0	1.000110	
1	0.034221	0.001280
2	0.000719	0.000077

Example 5.4: Example use of a sun-path diagram:

The solar zenith angle can also be calculated graphically using sun path diagrams such as shown below for the latitude of 41°N. This diagram is a convenient graphical way of representing Eqn. (5.3).

Example Sept. 21, at 9 a.m., azimuth = 122°; $\theta_{\odot}$ ~ 57° (elevation angle 33°)



An example of a sun path diagram for a latitude near that of Ft. Collins.

5.4 Variability of Solar Flux Outside the Atmosphere

There are two main causes for the variability of $Q_{\odot}$, and these manifest themselves in very different ways over an enormous range of time scales. The first has to do with changes in the radiation output of the sun itself and the second has to do with changes in astronomical factors that influence how this output is received at Earth.

(a) The Flickering Sun

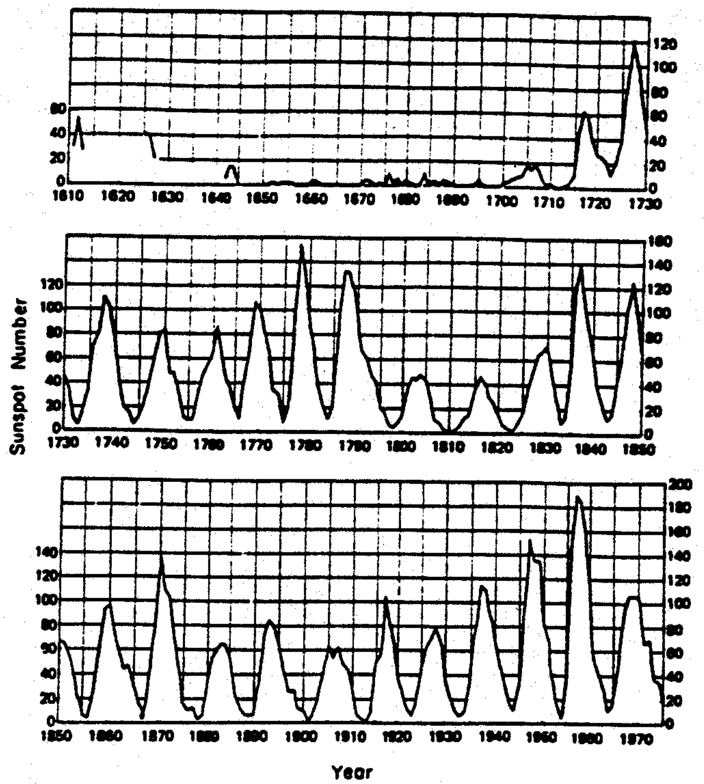
(Reference: Foukal: The Variable Sun, Scientific American, Feb, 1990). The emission from the sun varies in time. Large changes in coronal activity are well established giving rise to changes in coronal UV emission and microwave/radiowave emissions. This variable output was indicated previously in Fig. 5.2b. However, the greater part of the solar emission comes from the photosphere where the magnitude of the variability is much less. The shorter term variabilities of solar output (order of 0.2%) over time scales of weeks is thought to be caused by passage of

- sunspots (dark spots on 'surface' of the sun) across disc

- faculae (bright spots) associated with the sun's magnetic activity that accompanies sunspots.

Long-term variability can also be identified with the solar cycle. The output decreased by about 0.1% between the peak in 1981 and its minima in mid 1986 (Fig. 5.7a). The sun grew more luminous as the sunspots grew larger—area covered by bright faculae outweighs the increase in area by dark sunspots.

(a)



(b)

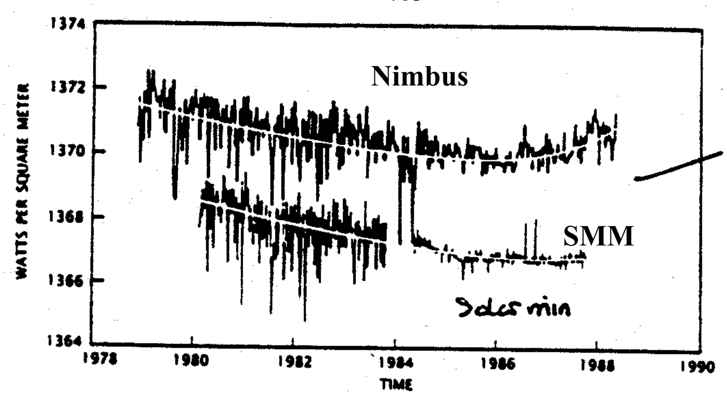


Fig. 5.7 *(a) Flickering of the sun was recorded by radiometers on two satellites, Nimbus 7 (blue) and Solar Maximum Mission (red). Short-term decreases in solar output produced the sharp spikes in the SMM data, and most of those seen in the Nimbus 7 data, which also included some instrument noise. On the average (yellow line) the sun shone brightest at the time of maximum sunspot activity. Apparently the greater number of bright faculae at maximum activity outweighed the effect of dark spots. (b) Solar cycle manifests itself in the changing number of spots on the sun's visible surface (left). The dearth of spots between about 1645 and 1715, known as the Maunder minimum, appears to coincide with an era of unusually cold weather.*

(b) Astronomy

The three main astronomical factors that govern the radiation received on a flat surface, namely δ, orbital eccentricity and axial precession all vary in a regular manner as shown in Fig. 5.8a and b. Vernekar (1972: Long period global variations of incoming solar radiation, *Meteorological Monographs,* **12**, No. 24) shows how the solar irradiance varies in time and as a function of latitude. An example is given in Fig. 5.9a. The upper panel shows the changes in the radiation for the NH winter and the lower for the SH winter. The main point is that the distribution of irradiance is altered significantly but the global and annual average is not (units quoted are in Ly day^{-1}, compare these numbers with those presented in Fig. 5.6 to gain some idea of the percentage change in $Q_{\odot}$. The characteristics of the variabilities in the astronomical factors appear in climate records (Fig. 5.9b). Another reference of relevance is that of Berger (1987: Long-Term variations of Daily insolation and quaternary climatic changes, *J Atmos. Sci.*, **35**, 2362-2367).

(a)

(b)

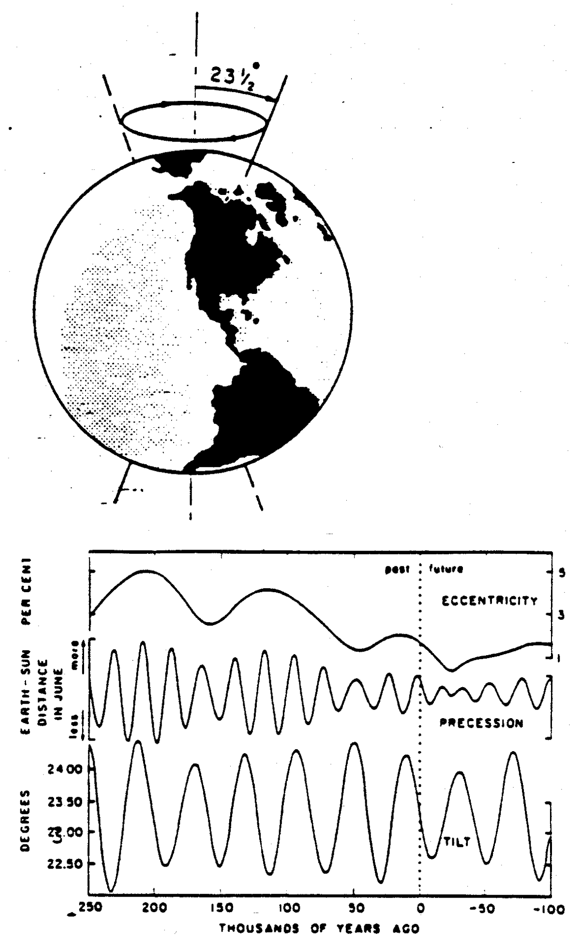
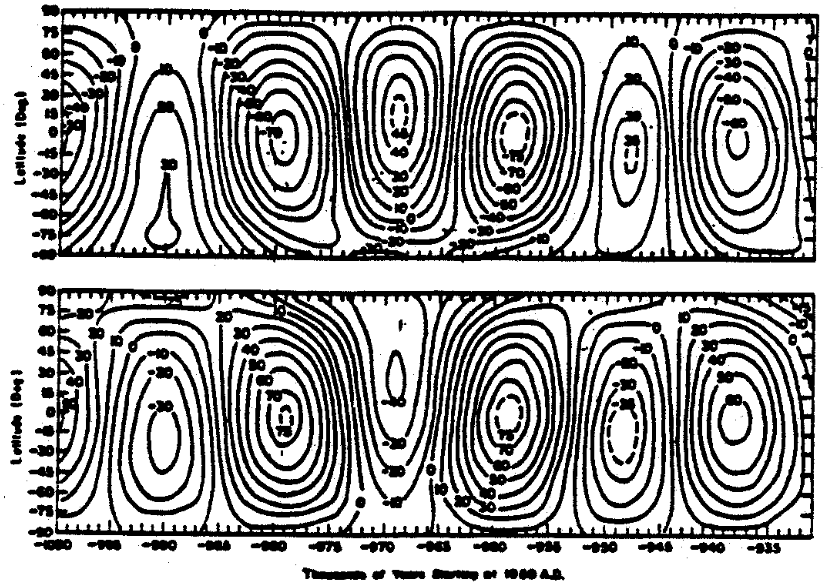


Fig. 5.8 *(a) Precession of the earth. Owing to the gravitational pull of the sun and moon on the equatorial bulge of the earth, its axis of rotation moves slowly around a circular path and completes one revolution every 26,000 years. Independently of this cycle of axial precession, the tilt of the earth's axis (measured from the vertical) varies about 1.5° on either side of its average angle of 23.5°. (b) Changes in eccentricity, tilt, and precession. Planetary movements give rise to variations in the gravitational field, which in turn cause changes in the geometry of the Earth's orbit. These changes can be calculated for past and future times. (Data from A Berger.)*



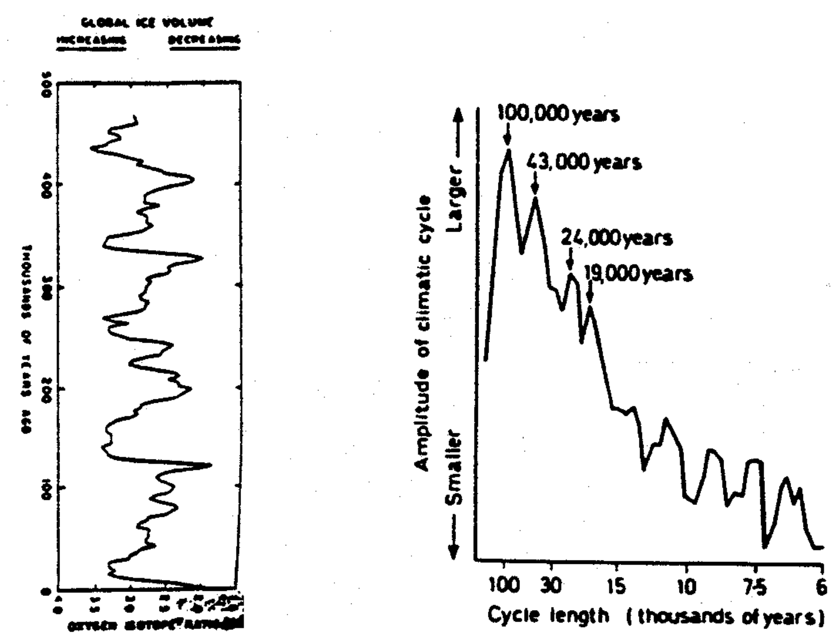


Fig. 5.9 *(a) Variation in $\Delta Q_{\odot}$ (ly day^{-1} as a function of latitude and time measured from 1950 AD). (b) Time series of isotopic measurements (these reflect global ice volume) from two Indian Ocean cores (upper panel) and the spectrum of this variation showing the imprint of different climatic cycles in the isotopic record—these seem to support predictions of the Milankovitch theory. (Data from J.D. Hays et al. 1976.)*

5.6 Problems

Problem 5.1

Given the following characteristics for Mars and Jupiter:

Sun Diameter = 1,390,600 km
Mars Diameter = 6860 km
Jupiter Diameter = 143,600 km
Sun—Mars Distance = 228 X 10^6 km
Sun—Jupiter Distance = 778 X 10^6 km
Mars Albedo = 16%
Jupiter Albedo = 73%
Solar Output = 6.2 kw cm^{-2}

Calculate for each planet

(a) Solar Constant
(b) Equivalent Blackbody Temperature
(c) Wavelength of Maximum Emission
(d) Solid Angle Subtended by the Sun

Problem 5.2

Calculate the radiative equilibrium planetary temperatures for earth assuming albedos of 0.2, 0.3, 0.4, and 0.5.

Problem 5.3

Calculate the net longwave power per unit area gain/loss of a grass surface at 2°C when under a clear sky with an effective temperature of -30°C, and when under a tree with an effective temperature of 5°C.

Problem 5.4

Find the wavelength at which the incoming solar irradiance at the top of the earth's atmosphere is equal to the outgoing terrestrial irradiance. Assume the sun and earth to be emitting as blackbodies at 6000°K and 255°K, respectively.

Problem 5.5

Show that for an isothermal, surface-atmosphere system the upward infrared irradiance is invariant with height.

Problem 5.6

(a) Derive a relationship between solar irradiance and the distance (d) between the sun and the observation. (Assume $R_{.sun} << d$).
(b) Does the radiance obey the same relationship?

Problem 5.7

If the average output of the sun is 6.2 kw cm^{-2}, the radius of the sun is 0. 71 X 10^6 km, the distance of the sun from the earth is 150 X 10^6 km, and the radius of the earth is 6.37 X 10^3 km, what is the total amount of energy intercepted by the earth?

Problem 5.8

Calculate the solar zenith angle and azimuth angle for the following dates, locations, times:

Date	Latitude	Longitude	Time
1 Jan.	0°	20° W	1200 GMT
22 Mar.	30° N	180° W	1800 GMT
1 July	45° S	90° W	0900 Local sun time
1 Nov.	60° S	35° E	1000 Local sun time
1 Dec.	75° N	45° W	0600 Local sun time

Note: Reference your azimuth angle from the south such that east of south is negative and west of south is positive.

Problem 5.9

(a) Derive a simple expression for the elevation angle of the sun at local noon as a function of date and latitude.
(b) Derive an expression for time of sunrise as a function of date and latitude.

Problem 5.10

Calculate the azimuth angle of sunrise at 40° N on June 21. Sketch the sun-earth geometry and interpret your results.

Problem 5.11

(a) The eccentricity of the earth's orbit is 0.01673. What would be the percentage variation in the irradiance at the top of the atmosphere due to this eccentricity from time of apogee to time of perigee?
(b) On which dates would you observe these min max values?

Problem 5.12

An aircraft is being used to measure the surface albedo for a certain region. The downward irradiance is measured with a pyranometer to be 750 Wm^{-2} and the upward irradiance is measured to be 250 Wm^{-2}. The angle of attack (angle between the horizontal and the plane of the wing) is known to be 4°. If the plane is flying due west at latitude 0° on Julian day 80, at 1500 local solar time, and assuming that the total radiation striking the upward looking sensor is 40% direct and 60% pure diffuse, calculate the albedo of the underlying target.

Problem 5.13

In Table 5.1 the latitudinal distribution of both the fractional day length and mean cosine of the solar zenith angle are listed for the Northern Hemisphere. Calculate the matching values of these quantities for the June 21 solstice for the Southern Hemisphere using the equivalent 9 (Southern) latitudes. Use these values to provide the latitudinal distribution of the daily solar insolation for this date and compare your results with those of Figure 5.6a of your notes.