AT622, Homework # 4

Due no later than Monday May 11, 2015

1. a) Using Petty eqn. 11.32* (the single-scattering approximation), plot the downwelling sunlight at the surface for a clear atmosphere with an overhead sun as a function of viewing zenith angle θ for blue light (450 nm) and red light (650 nm). Assume that Fsun(blue) = Fsun(red) = 1.0 for simplicity, and that both wavelengths are non-absorbing. Recall the Rayleigh scattering optical depth for a clear atmosphere is given approximately by 0.00877 $\lambda^{-4.05}$, where λ is the wavelength in μm. **Note: Petty 11.32 has an overall minus sign error. Also remember his sign convention, so both μ and μ_0 are negative in his convention. The equation in the powerpoint slides online is correct.*

b) How do the intensity and apparent color of the light change as you look more toward the horizon? For color, examine the fraction of the light that is red vs. blue.

c) What happens to the apparent color if you were to use the very approximate version of the equation, Petty 11.33? Which result (b or c) is more consistent with your experience?

2. Using the single scattering approximation, we will calculate the flux reflectivity of a thin cirrus cloud, and then add in a Rayleigh atmosphere and see what happens.

a) Assuming an cloud ice water path of 20 g/m^2, and an effective radius of 50 um, calculate the cirrus optical depth at visible wavelengths (Petty eqn 7.74). Assume the density of ice is 0.9 that of water, and that the ice particles are spherical (this is really not true, but we hope the "effective radius" bit takes care of this problem).

b) Now calculate the flux reflectivity of the cloud. Hint: use Petty eqn 13.63, which assumes the 2-stream approximation and that the cloud is non-absorbing (ssa=1). Assume a typical cloud asymmetry parameter of 0.85.

c) Qualitatively, how does the flux reflectivity of the cloud change is the asymmetry parameter decreases? Explain why.

3. a) Using Rayleigh theory, find Q_e, Q_s, Q_a, and the single scattering albedo of the *ammonium sulfate* aerosol at visible (500nm) and UV (350 nm) wavelengths, assuming small particles of diameter 0.1 micron. Ammonium sulfate indices of refraction have been measured to be (1.52,0) in the visible and (1.53, 0.004) in the UV. Is sulfate aerosol more absorbing or more scattering in the UV? Finally, if a 500-m thick layer of this aerosol is observed to have a moderate optical depth of 0.1 in the visible, compute the number density of aerosols (in particles per cubic cm) that yields the observed optical thickness. Assume the particles all have this same diameter of 0.1 micron. Typical number densities are a few thousand per cm^3 in relatively clean air to many hundreds of thousands per cm^3 in polluted air.

b) You will find an extremely high concentration, even for this moderate optical depth. Briefly explain why, using the fact that realistic distributions of aerosols will contain some larger and some smaller particles, coupled with your knowledge of how the Rayleigh scattering cross section per particle varies with particle size. For example, if Rayleigh theory held (it almost does), what concentration would yield the same optical depth at λ=500 nm if the diameter of the particles was 0.2 microns instead of 0.1?

4. Petty Problem 12.7

5. Petty Problem 12.9

5. We learned in class that heating rates in the atmosphere result from changes in the net fluxes in a certain layer of the atmosphere (see Eqn 10.54 in Petty).

$$\boxed{\mathcal{H} \equiv -\frac{1}{\rho(z)C_p}\frac{\partial F^{\text{net}}}{\partial z}(z)\,,} \tag{10.54}$$

This equation can be expanded to the form of Eqn 10.65:

$$\begin{aligned}
\mathcal{H}(z) = \frac{1}{\rho(z)C_p}\Bigg\{ & \\
& -\left[F_i^{\uparrow}(0) - \Delta\tilde{\nu}_i\pi\overline{B}_i(z)\right]\frac{\partial\mathcal{T}_i(0,z)}{\partial z} && (A) \\
& +\left[F_i^{\downarrow}(\infty) - \Delta\tilde{\nu}_i\pi\overline{B}_i(z)\right]\frac{\partial\mathcal{T}_i(z,\infty)}{\partial z} && (B) \\
& -\Delta\tilde{\nu}_i\pi\int_z^{\infty}\left[\overline{B}_i(z') - \overline{B}_i(z)\right]\frac{\partial^2\mathcal{T}_i(z,z')}{\partial z'\partial z}\,dz' && (C) \\
& -\Delta\tilde{\nu}_i\pi\int_0^{z}\left[\overline{B}_i(z') - \overline{B}_i(z)\right]\frac{\partial^2\mathcal{T}_i(z',z)}{\partial z'\partial z}\,dz' && (D) \\
\Bigg\}. &
\end{aligned} \tag{10.65}$$

which is explained by four terms (remember we are ignoring scattering):

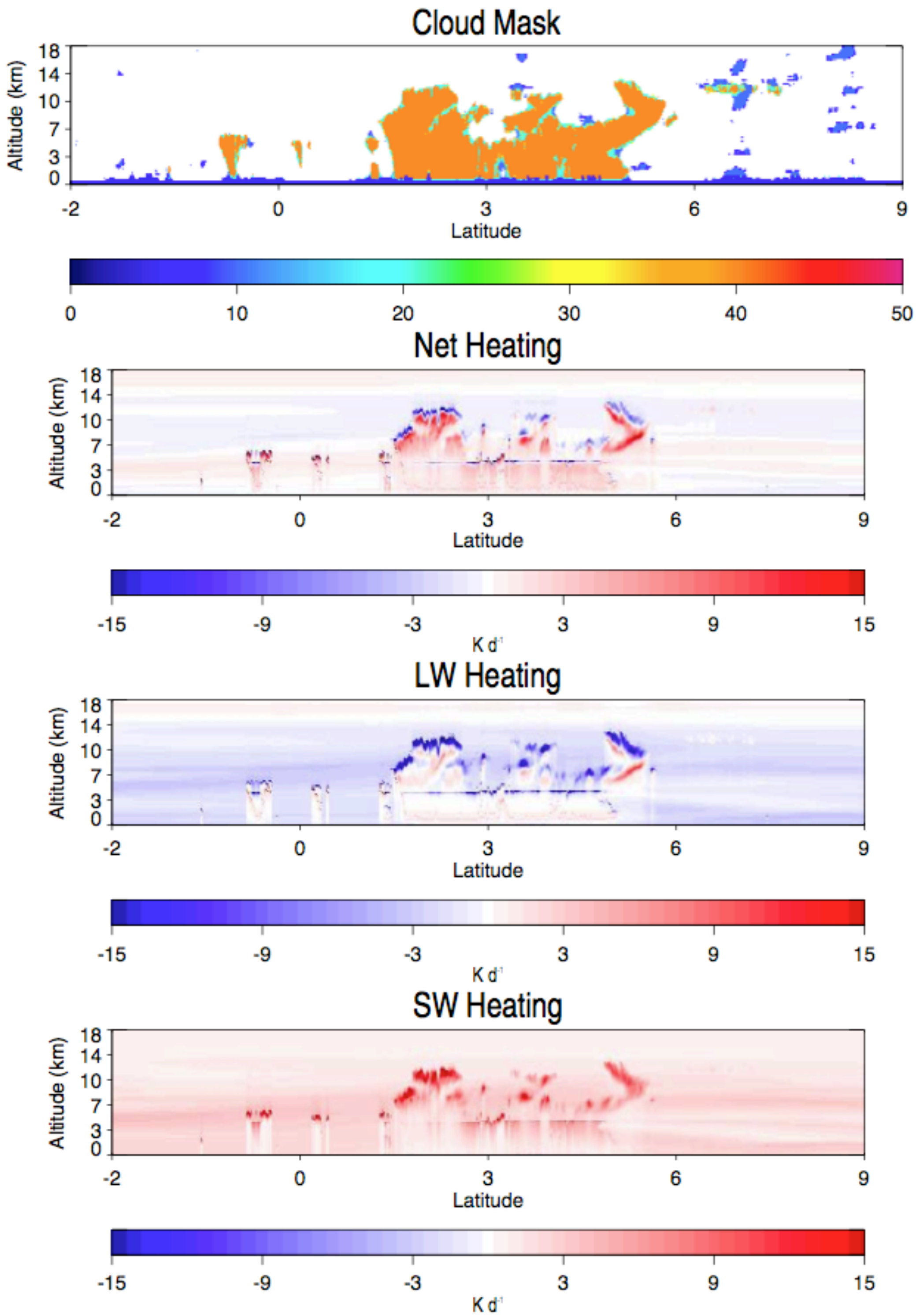
A) Coupling to the surface
B) Coupling to the top of the atmosphere
C and D) Coupling to the surround layers.

Based on class discussion & the book, explain the following:

a) Which terms are most important for solar wavelengths and which are most important for thermal IR wavelengths?
b) When are terms C and D considered negligible?

Equation 10.65 can be used to explain heating rates in a real world scenario. The attached figure displays heating rates calculated by CloudSat's 2B-FLXHR (Fluxes and Heating Rates) algorithm from a convective cloud retrieved by CloudSat (centered around 3° N and heating in units of K/day). Use the terms in Equation 10.65 to briefly explain the following:

c) Why do we see such a large gradient in net heating at cloud top?
d) Why is there a slight increase in LW heating at cloud base (near the surface) ?
e) Knowing that there is a lot of cloud water in the convective cloud, why do we see near zero heating in the LW from around 2-3 km?
f) How would you expect the heating rates to change if a cirrus cloud were added at ~15km?


Cloud Mask
Altitude (km)
18 14 10 7 3 0
-2 0 3 6 9
Latitude
0 10 20 30 40 50
Net Heating
Altitude (km)
18 14 10 7 3 0
-2 0 3 6 9
Latitude
-15 -9 -3 3 9 15
K d⁻¹
LW Heating
Altitude (km)
18 14 10 7 3 0
-2 0 3 6 9
Latitude
-15 -9 -3 3 9 15
K d⁻¹
SW Heating
Altitude (km)
18 14 10 7 3 0
-2 0 3 6 9
Latitude
-15 -9 -3 3 9 15
K d⁻¹