Homework 2 – ATS622 Spring 2015.
Due Friday Feb 20 end-of-day

1. Petty 6.5, but for part c you do not need to plot the Planck function. *Do* estimate the fraction of light in the visible band.

2. Petty 6.8. In particular, what condition on $x=hc/(\lambda k_B T)$ must be satisfied for the Rayleigh-Jeans approximation to be accurate to better than 1%?

3. Petty 6.11.
4. Petty 6.12 (should be very easy based on in-class discussion.)
5. Petty 6.14
6. Petty 6.18
7. Petty 6.20
8. Petty 6.26

9. In this problem we'll approximate the greenhouse effect of the atmosphere with a simple energy balance approach. Assume an idealized isothermal earth atmosphere, which allows all solar radiation to pass through, but absorbs all thermal infrared radiation. This would be a "perfect" greenhouse, at least for an isothermal atmosphere. Further assume that the surface of the earth has a shortwave albedo of 0.3 and longwave emissivity of unity. Write down an equation for the equilibrium surface temperature of the earth as well as for the atmosphere under these assumptions. What are these two temperatures using a solar constant of 1370 W/m^2 and the emissivities and albedos given?

Since we know the actual mean surface temperature of the earth is 288 K, what would the emissivity of the atmosphere need to be to allow this? What would the mean temperature of the atmosphere change to in this case?

10. The orbital parameters of the earth change over time. The eccentricity of the earth's orbit ε is currently 0.0167, but changes from values near 0 to about 0.04 over tens of thousands of years. The tilt of the earth's spin axis relative to the orbital plane around the sun is called the *obliquity.* This is currently 23.45°. However, it varies from a minimum value of about 22° to a maximum of about 24.5° with a mean period of 41,000 years. In this problem we'll calculate daily, zonally averaged top-of-atmosphere solar insolation, which depends on the solar *declination* (latitude of an overhead sun) and distance to the sun. An approximate formula for $(a/r)^2$ as a function of time, where a=1 AU (the distance from the center of the earth's orbital ellipse to perihelion) and r is the distance from the earth to the sun, is:

$$(a/r)^2 = 1 + 2\varepsilon \cos(2\pi\,(JD-3)/365)$$

where ε is the earth's orbital eccentricity and JD is the day-of-year (JD=1 means January 1). JD=3 corresponds to the current JD of closest approach of the earth to the sun, the *perihelion*. This formula is valid for relatively low values of the eccentricity, which is nearly always satisfied for Earth.

An approximate formula for the solar declination δ[1] is

$$\delta \approx \text{-obliquity} * \cos(2\pi\ (JD+9)/365)$$

a. Using the equations above and from class, reproduce the contour plot of daily average insolation vs. both time of year and latitude as shown in Petty Fig 2.9.

b. Calculate the distribution of solar insolation as above, but now for an orbit with a lower obliquity of 22°. Plot the *difference* of the solar insolation for this new value of the obliquity (22°) minus that using the current value from part (a), averaged over a year, versus latitude. Based on this information, will the orbit with the lower obliquity tend to make ice ages more or less likely? Why? (Note that the daily-averaged summertime insolation in high northern latitudes is theorized to control the onsite of ice ages – see https://geosci.uchicago.edu/~archer/reprints/archer.2005.trigger.pdf)

[EXTRA CREDIT - only work on this if you're interested and have time!]

The heating & cooling of the moon.

Because of tidal locking, the length of a lunar day is P=27.3 Earth days, equal to its orbital period around the earth. Assume that at a certain location on the moon, the solar elevation angle (the angle between the horizon and the sun) is given approximately by 70 sin(2πt/P), where t is the time in days, and P is length of the lunar day.

Write down an equation for the net flux at the surface as a function of time, assuming a lunar shortwave albedo of 0.12 and a longwave emissivity of 1. Use this to calculate the heating rate of the surface as a function of time, assuming that the heat can instantaneous mix down to a soil depth of 10 cm, that the specific heat capacity of the lunar soil is 1000 J/(kg K), and the lunar solar density is 3000 kg/m^3.

Assuming that the temperature at time=0 is 254 K, plot the following quantities over the course of a few lunar days:

a. Incoming absorbed solar flux vs. time
b. Net Flux vs. time
c. Temperature vs. time

Discuss when the maximum daily temperature occurs, and how this seems to relate to the net radiation.

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[1] This formula ignores eccentricity. A much more exact formula is

$$\delta = \text{ArcSin}[\sin(\text{-obliquity}) * \cos(2\pi\ (JD+9)/365 + 2\varepsilon \sin(2\pi(JD-3)/365)\ ]$$