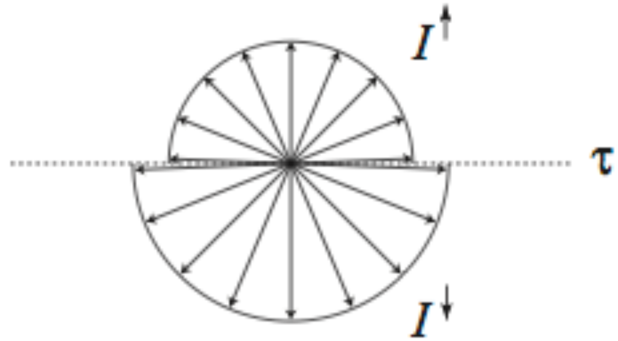
Single-layer reflectivity plots



$$I(\mu) = \begin{cases} I^{\uparrow} & \mu > 0 \\ I^{\downarrow} & \mu < 0 \end{cases}$$

$$\boxed{\frac{1}{2}\frac{dI^{\uparrow}}{d\tau} = (1-\tilde{\omega})I^{\uparrow} + \frac{\tilde{\omega}(1-g)}{2}(I^{\uparrow} - I^{\downarrow})\,,} \tag{13.16}$$

$$\boxed{-\frac{1}{2}\frac{dI^{\downarrow}}{d\tau} = (1-\tilde{\omega})I^{\downarrow} - \frac{\tilde{\omega}(1-g)}{2}(I^{\uparrow} - I^{\downarrow})\,.} \tag{13.17}$$

Boundary Conditions

At this point we are nearly finished — only two coefficients in our solution remain undetermined. To find these coefficients, we need to supply two boundary conditions appropriate to the problem we wish to solve. Let's choose the following:

$$I^{\uparrow}(\tau^*) = 0 \quad ; \quad I^{\downarrow}(0) = I_0\,, \tag{13.36}$$

$$\boxed{I^{\uparrow}(\tau) = \frac{r_\infty I_0}{e^{\Gamma\tau^*} - r_\infty^2 e^{-\Gamma\tau^*}}\left[e^{\Gamma(\tau^*-\tau)} - e^{-\Gamma(\tau^*-\tau)}\right]\,,} \tag{13.39}$$

$$\boxed{I^{\downarrow}(\tau) = \frac{I_0}{e^{\Gamma\tau^*} - r_\infty^2 e^{-\Gamma\tau^*}}\left[e^{\Gamma(\tau^*-\tau)} - r_\infty^2 e^{-\Gamma(\tau^*-\tau)}\right]\,.} \tag{13.40}$$

We adapt (13.39) and (13.40) to the case of a semi-infinite cloud simply by letting $\tau^* \to \infty$, which gives us

$$I^{\uparrow}(\tau) = I_0 r_\infty e^{-\Gamma\tau} , \tag{13.41}$$

$$I^{\downarrow}(\tau) = I_0 e^{-\Gamma\tau} . \tag{13.42}$$

$$\boxed{r_\infty \equiv \frac{\sqrt{1-\tilde{\omega}g} - \sqrt{1-\tilde{\omega}}}{\sqrt{1-\tilde{\omega}g} + \sqrt{1-\tilde{\omega}}} .}$$

“Semi-infinite Cloud”

$$r_\infty \equiv \frac{\sqrt{1-\tilde{\omega}g}-\sqrt{1-\tilde{\omega}}}{\sqrt{1-\tilde{\omega}g}+\sqrt{1-\tilde{\omega}}}\,.$$

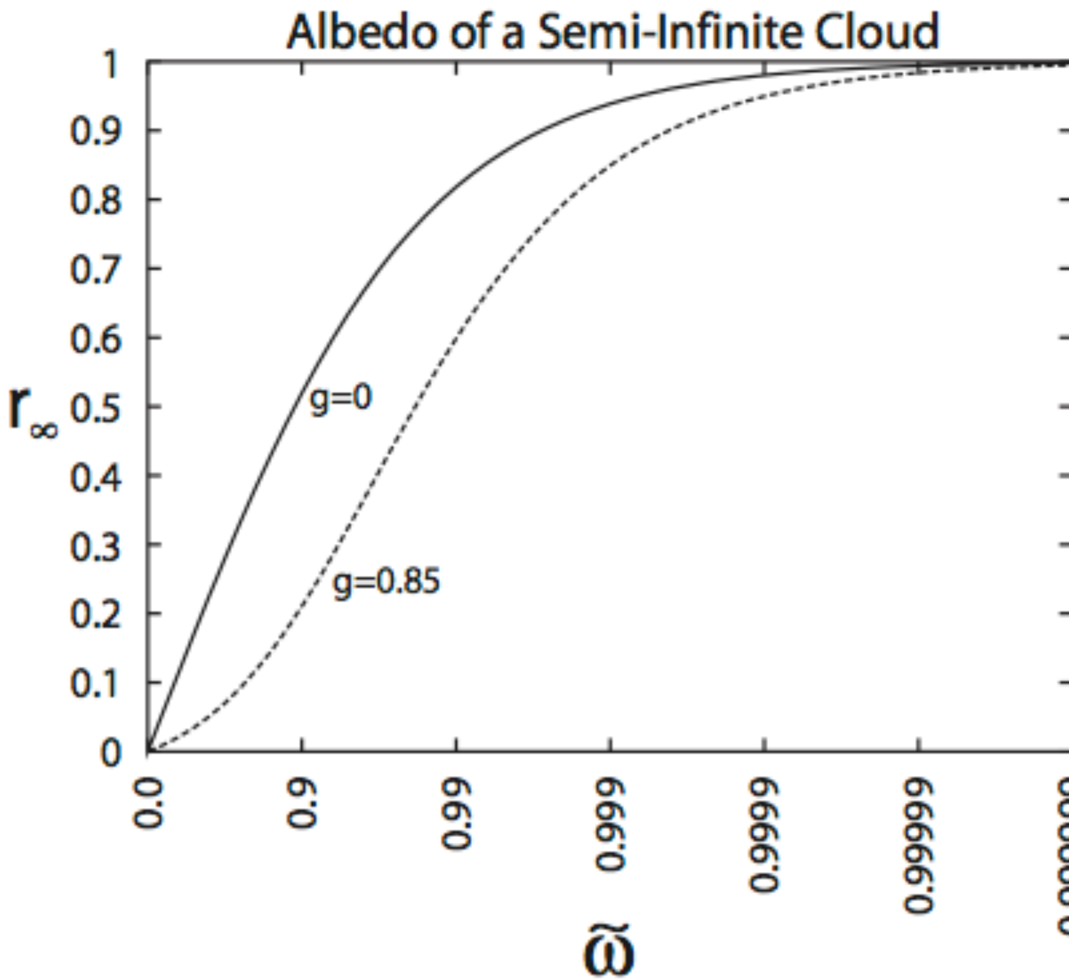


Fig. 13.4: The albedo of a semi-infinite cloud, as computed from (13.45).

13.5.1 Albedo, Transmittance, and Absorptance

Starting from our two-stream solutions (13.39) and (13.40), we find that the general expression for the albedo for the case that $\tilde{\omega} < 1$ is

$$r = \frac{r_\infty \left[e^{\Gamma\tau^*} - e^{-\Gamma\tau^*}\right]}{e^{\Gamma\tau^*} - r_\infty^2 e^{-\Gamma\tau^*}}, \qquad \tilde{\omega} < 1\,, \tag{13.65}$$

and the total transmittance is

$$t = \frac{1 - r_\infty^2}{e^{\Gamma\tau^*} - r_\infty^2 e^{-\Gamma\tau^*}}, \qquad \tilde{\omega} < 1\,. \tag{13.66}$$